Selfish Routing and Congestion Games: Towards a Game-Based Analysis of the Internet

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The application of Game Theory to networks, particularly to routing in networks, is gaining a growing amount of interest in the computer science community. The reason is that large-scale communication networks like the Internet cannot be managed by a central authority. This motivates the analysis of network traffic using models from Game Theory in which single packets or data streams are managed by selfish agents trying to minimize their individual latencies. In this tutorial, we give an overview over the recent work in this area. We introduce the investigated models and concepts from game theory, present some of the most remarkable results obtained in this area, and point out some relevant open problems. In particular, we cover the following topics.

Price of Anarchy: One of the common assumptions in this research area is that the routing process should arrive into a Nash equilibrium in which no network user has an incentive to change its strategy. It is well known (and easy to see) that Nash equilibria do not always optimize the overall performance of the system. The so-called *price of anarchy* or *coordination ratio* measures the degradation of the performance due to uncoordinated behavior in form of a comparison between the latency experienced in a worst case Nash equilibrium and the latency in an optimal assignment. We present the models and results from [1, 5, 7] and others.

Algorithms for computing Nash equilibria: A classical result by Rosenthal [6] shows that congestion games, one of the kind of games underlying selfish routing, admit pure equilibria. In fact, Rosenthal's result is based on an elegant potential function argument proving that any sequence of improvement steps performed by the agents leads to a Nash equilibrium. Unfortunately, however, this potential function argument only yields pseudopolynomial time algorithms for computing such equilibria. We present Rosenthal's analysis as well as various recent results [4, 2, 8] concerning the complexity of computing pure and mixed equilibria for congestion and other routing games.

Evolution of selfish routing: Classical Game Theory is based on fully rational behavior of agents and global knowledge of all details of the game under study. For routing games in large networks like the Internet, these assumptions are obviously far away from being realistic. Evolutionary Game Theory takes a different attempt to explain why large populations of agents may or may not "converge" towards equilibrium states: Agents try to improve their situation by replicating the locally observed strategies of other, more successful agents. We present an evolutionary model for selfish routing and study questions concerning the stability of equilibria and the speed of convergence towards them [3, 9].

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